Serie 03 - Solution

Preamble

Space Charge Region

As discussed during the lecture, in an intrinsic semiconductor, each electron promoted to the conduction band leaves a hole in the valence band. Both of these carriers can move more or less freely within the semiconductor. Even though theoretically, the random motion of free carriers can generate localized regions containing slightly more holes or electrons, the electric field generated by this charge difference will immediately attract the needed charge and neutralize these regions. Therefore, such phenomena are never observed in practice.

In the case of a uniformly doped semiconductor, the concept remains similar. For example, when a donor dopant is ionized, the atoms donate one of their electrons to the conduction band, generating a free electron and becoming slightly positive themselves. Unlike the electrons or holes generated by dopant ionization, the dopants themselves, being part of the crystal structure, cannot move easily. Nevertheless, the principle remains the same as for intrinsic semiconductors: if theoretically, the random motion of free carriers can generate localized regions with a slight charge, these regions will be immediately neutralized by the electric field generated.

In the case of an n-type non-uniformly doped semiconductor, the electron density depends on the donor concentration, but not only. Similar to many other phenomena, free carriers tend to flow towards regions where the carrier density is lower. The formulas that describe this flow (F_n and F_p) are as follows:

$$F_n = -D_n \frac{dn}{dx} \qquad F_p = -D_p \frac{dp}{dx} \tag{1}$$

This flux of carrier diffusion will create what is known as the diffusion current. In our case, electrons flow towards regions with lower electron density, while the donors, being part of the crystal structure, remain immobile. Consequently, the region from which the electrons flow has a higher concentration of ionized donors (positive) compared to electrons (negative), resulting in a positive charge in that region. Conversely, the region into which the electrons flow has a higher electron (negative) concentration than the ionized donor (positive) concentration, creating a negatively charged region. These charged regions generate an electric field that counteracts the diffusion current, leading to a drift current. A similar reasoning can be applied to p-type semiconductors.

The best example of such a phenomenon is, of course, the p-n junction. In all cases, the overall structure must remain neutral at thermal equilibrium. This is evident from the preceding paragraph, but it is also essential to prevent the generation of an electric field within the structure.

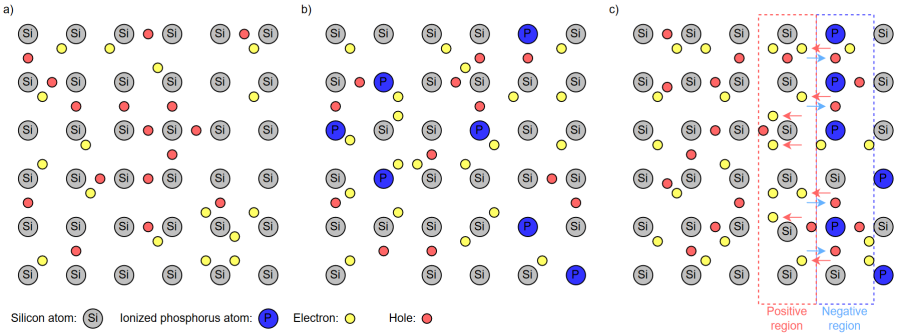

Figure 1: a) Intrinsic semiconductor. b) Uniformly doped n-type semiconductor. c) Non-uniformly doped n-type semiconductor.

Overall charge neutrality

During the formation of the space charge region, charges are moved around, but none appear or disappear. This leads to an overall neutrality of the PN-junction. In other words, even if space charge regions are created, the total amount of positive and negative charge will be zero. By applying Maxwell's equations (Gauss's law), we can therefore say that outside of the space charge region, no electric field will exist. Another way to see this is that if the overall structure of the PN-junction is not neutral, an electric field will be created, attracting any possible charge in the outside world to cancel out these non-balanced charges.

Given constants

$n_i(Si) = 1.5 \cdot 10^{10}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$k = 8.62 \cdot 10^{-5}\,[eV/K]$
$q = 1.60 \cdot 10^{-19}\,[C]$
$\epsilon_0 = 8.85 \cdot 10^{-14}\,[F/cm]$
$\epsilon_{Si} = 11.7 \cdot \epsilon_0$

Exercise 01

A silicon bar is doped with acceptor dopants following a profile shown in Fig. 2. The dopant density, N_a, increases quasi-monotonically from $N_{amin} \ll n_i$ at $x = 0$ to $N_{amax} = \text{cst}$ at $x = L$. It is considered that $N_a = n_i$ at $x = L/2$, the dopant concentration saturates to a constant value near $x = L$, the donor concentration is $N_d = 0$ throughout the bar, and finally, thermal equilibrium is assumed.

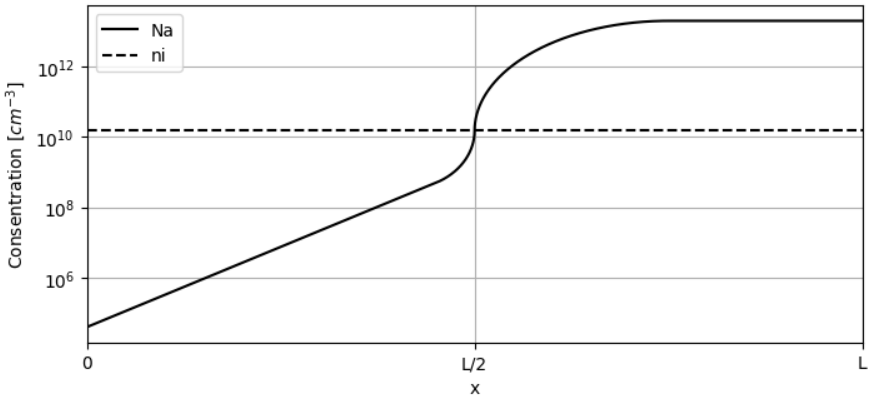

Figure 2: Non-uniform doping profile.

Solution

The first thing to do here is to roughly draw the electron n_0 and hole concentration p_0.This drawing is relatively easy to perform, firstly we will not take into account the diffusion and drift current and, as this dopant is a donor we will focus our self on an approximative hole concentration p_a. In this condition, we cans use the approximations developed in the **To Go Further** subsection of **Exercise 02** in the correction of **Series 01**:

$$p_a \approx N_a^- - N_d^+ \quad \text{, if} \quad \left(N_d^+ - N_a^-\right) \gg n_i \tag{2}$$

and:

$$p_a \approx n_i \quad \text{, if} \quad \left|N_a^- - N_d^+\right| \ll n_i \tag{3}$$

This two approximations lead to the following result, when $N_a \ll n_i$ the hole concentration p_a follow n_i and when $N_a \gg n_i$, p_a follow N_a.

Now we will consider the contribution of both diffusion and drift currents. In this case, it is easier to think about the flow of carriers. In other words, some of the holes from the higher concentration will be added to the lower concentration. Based on the equations that govern diffusion and drift currents, we can

also conclude that the electron and hole concentrations must be continuous.

Now that we have the hole concentration, the electron concentration is easy to deduce: $n_0 = n_i^2/p_0$. And since we are in logarithmic representation, we can simply mirror the hole concentration with the n_i line.

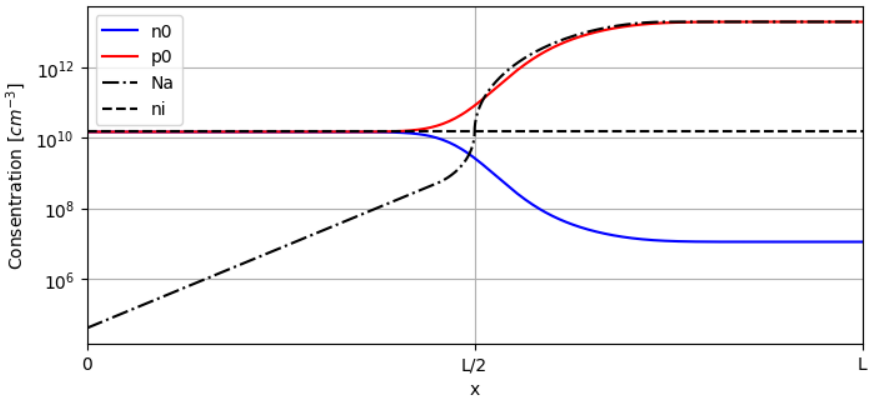

Figure 3: Electron and hole concentration.

Questions

Choose the correct answer to the following questions:

Q1. Where is the maximum hole concentration reached in this structure?

a) $x = 0$
b) $0 < x < L/2$
c) $x = L/2$
d) $L/2 < x < L$
e) $x = L$

Sol: **e)** $x = L$ can be deduced directly from Fig. 3. The further you are from the non-uniform doping region, the closer you are to the target value of $p(x) = N_a$ if $N_a \gg n_i$.

Q2. Where is the maximum electron concentration reached in this structure?

a) $x = 0$
b) $0 < x < L/2$
c) $x = L/2$
d) $L/2 < x < L$
e) $x = L$

Sol: **a)** $x = 0$, can be deduced directly from Fig. 3:

- The further you are from the non-uniform doping region, the closer you are to the target value of $p(x) = n_i$ if $N_a \ll n_i$.
- Even though the acceptors in such concentration have a negligible effect, they do not have no impact.

Q3. In which direction is the diffusion current of holes oriented?

a) $-\vec{x}$
b) $\vec{x}$
c) There is no diffusion current of hole.

Sol: **a)** $-\vec{x}$

- You can use the diffusion current discussed during the course: $J_p^{diff} \propto -dp/dx$
- The holes diffuse from the higher concentration to the lower one (in the direction of $-\vec{x}$). Since the charge of the holes is positive, the current is oriented in the direction of the diffusion (in the direction of $-\vec{x}$).

Q4. In which direction is the drift current of holes oriented?

a) $-\vec{x}$
b) $\vec{x}$
c) There is no drift current of hole.

Sol: **b)** $\vec{x}$

- As explained in the preamble of this series, at thermal equilibrium, the drift current is opposite in direction to the diffusion current.
- The diffusion flow generates a positive region on the left side and a negative region on the right side. The electric field is oriented in the direction of $\vec{x}$, and as a result, the drift current follows the direction of the electric field.

Q5. In which direction is the diffusion current of electron oriented?

a) $-\vec{x}$
b) $\vec{x}$
c) There is no diffusion current of electron.

Sol: **a)** $-\vec{x}$

- You can use the diffusion current discussed during the course: $J_n^{diff} \propto dn/dx$
- The electrons diffuse from the higher concentration to the lower one (in the direction of $\vec{x}$). Since the charge of the electrons is negative, the current is oriented in the reverse direction of the diffusion (in the direction of $-\vec{x}$).

Q6. In which direction is the drift current of electron oriented?

a) $-\vec{x}$
b) $\vec{x}$
c) There is no drift current of electron.

Sol: **b)** $\vec{x}$

- As explained in the preamble of this series, at thermal equilibrium, the drift current is opposite in direction to the diffusion current.
- The diffusion flow generates a positive region on the left side and a negative region on the right side. The electric field is oriented in the direction of $\vec{x}$, and as a result, the drift current follows the direction of the electric field.

Q7. In which direction is the electric field oriented?

a) $-\vec{x}$
b) $\vec{x}$
c) There is no electric field.

Sol: **b)** $\vec{x}$

- The electric field always follows the direction of the drift current.
- The diffusion flow generates a positive region on the left side and a negative region on the right side. The electric field is oriented in the direction of $\vec{x}$.

Q8. At which point in the structure is the internal electric potential maximum in absolute value? Considering that $\phi(x = 0) = 0$.

a) $x = 0$
b) $0 < x < L/2$
c) $x = L/2$
d) $L/2 < x < L$
e) $x = L$

Sol: **e)** $x = L$. The definition of electric potential is as follows:

$$\phi = -\int \mathbf{E} \cdot d\mathbf{l} \tag{4}$$

In our case, the electric field does not change direction, and if $\phi(x = 0) = 0$, the electric potential monotonically decreases until $x = L$. Therefore, the maximum absolute value of the electric field is at $x = L$.

Q9. At which point in the structure is the internal electric field maximum in absolute value?

a) $x = 0$
b) $0 < x < L/2$
c) $x = L/2$
d) $L/2 < x < L$
e) $x = L$

Sol: **c)** $x = L/2$. The drift current is linearly dependent on the electric field. The maximum electric field coincides with the maximum drift current. The drift current is opposite in direction but equal in magnitude to the diffusion current. The maximum electric field coincides with the maximum diffusion current. The diffusion current is linearly dependent on the derivative of the carrier concentration. The maximum electric field coincides with the maximum derivative of carrier concentration.

Q10. If the doping level N_a increases according to a new profile shown in Fig. 4, what happens to the absolute maximum internal electric potential?

a) It increase.
b) It decrease.
c) It doesn't change.

Sol: **a)** It increase.

- In this case, the concentration gradient increases, resulting in a higher number of charges flowing and consequently a higher electric field.
- In this case, as the concentration gradient increases, the diffusion current also increases. As a result, the drift current needs to increase to compensate for it, which in turn requires a higher electric field.

Q11. If the doping level N_a increases according to a new profile shown in Fig. 4, what happens to the absolute maximum internal electric field?

a) It increase.
b) It decrease.
c) It doesn't change.

Sol: **a)** It increase. The electric field increases and remains oriented in only one direction, therefore the electric potential also increases.

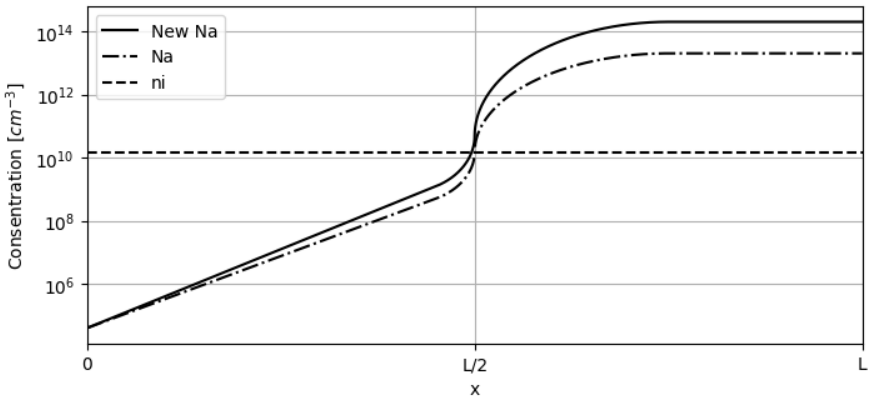

Figure 4: New non-uniform doping profile.

Exercise 02

We have a p-n junction at room temperature and thermal equilibrium. The p-type region of the junction is doped with $N_a = 10^{16}\left[cm^{-3}\right]$ and $N_d = 0\left[cm^{-3}\right]$. The n-type region of the junction is doped with $N_a = 0\left[cm^{-3}\right]$ and $N_d = 10^{17}\left[cm^{-3}\right]$. Use the depletion approximation for this exercice.

a) Calculate ϕ_n, ϕ_p, and ϕ_b the built-in potential.

b) Using the depletion approximation, draw and calculate the charge distribution in the p-n junction $\rho(x)$. Let x_{n0} and x_{p0} represent the unknown depletion widths on each side of the p-n junction.

c) Based on the calculated charge distribution, draw and calculate the electric field $E(x)$.

d) Based on the calculated electric field, draw and calculate the electric potential.

e) Use the built-in potential calculated at point **a)** and the potential calculated at point **d)** to determine the depletion widths on each side of the p-n junction, x_{n0} and x_{p0}.

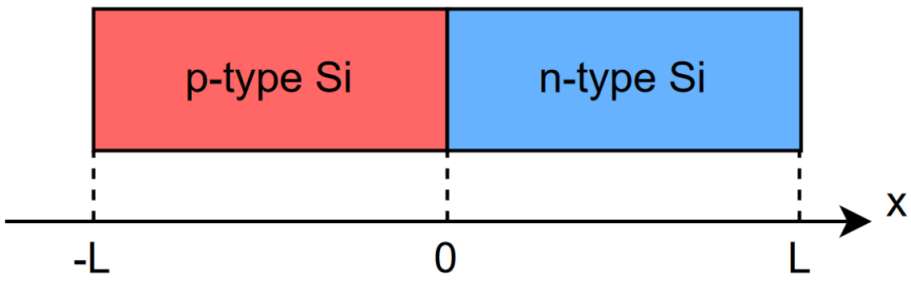

Figure 5: Exercise 02 p-n junction draw.

Solution A

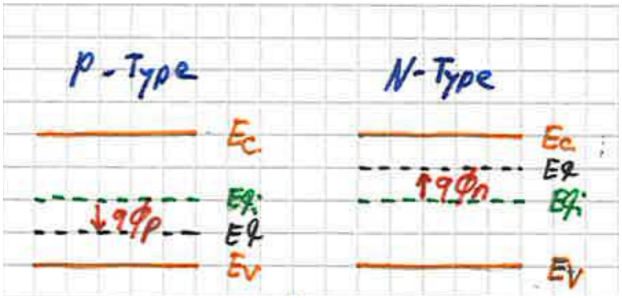

Figure 6: Illustration of ϕ_n and ϕ_p in a uniformly doped n and p-Type semiconductor.

As explained in the course and displayed in Fig. 6, ϕ_p and ϕ_n represent the voltage differences between the intrinsic Fermi level and the real Fermi level for the P-Type and N-Type regions of the PN-junction, respectively. To determine them, we simply applied the Boltzmann relation, which establishes a relationship between the electron and hole concentrations $n_0(x)$ or $p_0(x)$ and $\phi(x)$.

$$\phi_n = \frac{kT}{q} \ln\left(\frac{N_d}{n_i}\right) = 406\,[mV] \tag{5}$$

$$\phi_p = -\frac{kT}{q} \ln\left(\frac{N_a}{n_i}\right) = -347\,[mV] \tag{6}$$

Due to the diffusion mechanism, the real Fermi level cannot exhibit discontinuities along a semiconductor structure. When two structural pieces with different Fermi levels are joined together, a space charge region appears, generating a potential difference and bending the energy levels to achieve Fermi level continuity. Consequently, the built-in potential for an PN-junction can be viewed as a simple summation of ϕ_p and ϕ_n.This is illustrated in Fig. 7.

$$\phi_b = \phi_n - \phi_p = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 753\,[mV] \tag{7}$$

What is most important from a physical standpoint is the energy difference between levels. In semiconductor physics, for representing band diagrams, a convention often used is to, at thermal equilibrium, keep the Fermi level constant throughout the entire structure and to bend the other energy levels around it to ensure the correct energy differences. Example of such a representation is provided in Fig. 7.

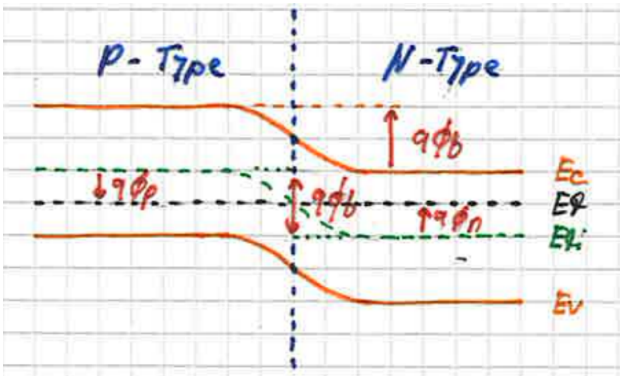

Figure 7: Band diagram of a standard PN-junction.

Note: Be very careful about the unit of the Boltzmann constant; the most common unit is $[J/K]$. Standard equations seen during the course use the Boltzmann constant with this unit. However, in some cases, including some series like this one, the Boltzmann constant provided may be in $[eV/K]$. In such cases, you have to convert the constant before use or adjust the formula accordingly. To convert from $[eV]$ to $[J]$, you multiply the value in $[eV]$ by the elementary charge q. In the case of the $[eV/K]$ unit, Eq. 5 becomes:

$$\phi_n = kT \ln\left(\frac{N_d}{n_i}\right) \tag{8}$$

Solution B

The charge density distribution is shown in Fig. 8. Using the depletion approximation, we can infer that all the holes from $-X_{p0}$ to 0 and all the electrons from X_{n0} to 0 have diffused toward each other and subsequently recombined, leaving behind the dopants in this region, which remain immobilized by the crystal. Consequently, in the P-Type regions, the ionized donors have captured an electron, resulting in a slightly negative charge. Thus, from $-X_{p0}$ to 0, we have a charge concentration of $-q$ multiplied by the dopant concentration N_a. A similar reasoning can be applied for the N-type region. This gives us:

$$\rho(x) = \begin{cases} 0 & , \quad x \in \left]\leftarrow; -X_{p0}\right] \\ -qN_a & , \quad x \in \left]-X_{p0}; 0\right] \\ qN_d & , \quad x \in \left]-0; X_{n0}\right] \\ 0 & , \quad x \in \left]X_{n0}; \rightarrow\right] \end{cases} \tag{9}$$

Solution C

The electric field is shown in Fig. 8. By applying Maxwell's equations (Gauss's law) in a one-dimensional case, we know that the electric field is simply the integration of the charge density along the x-axis divided by the dielectric constant of the current medium.

$$E\left(x\right)=\int_{-\infty}^{x}\frac{\rho\left(x\right)}{\epsilon}dx \tag{10}$$

In the case of charge density find before:

$$E\left(x\right)=\begin{cases}0 & , \quad x\in\left]\leftarrow;-X_{p0}\right]\\ E_0^- - \frac{qN_a}{\epsilon_{Si}}x & , \quad x\in\left]-X_{p0};0\right]\\ E_0^+ + \frac{qN_a}{\epsilon_{Si}}x & , \quad x\in\left]-0;X_{n0}\right]\\ 0 & , \quad x\in\left]X_{n0};\rightarrow\right]\end{cases} \tag{11}$$

In accordance with the **overall charge neutrality** explained in the **preamble** of this series, we understand that the electric field outside the space charge region is zero. Due to the nature of electric field integration and the uniform medium (having the same relative dielectric constant), we can establish that the electric field along the PN junction should be continuous. Therefore, we can utilize this knowledge to determine the integration constants E_0^- and E_0^+:

$$\lim_{x\rightarrow -Xp0^+}E\left(x\right)=0 \quad \Longrightarrow \quad E_0^-=-\frac{qN_a}{\epsilon_{Si}}X_{p0} \tag{12}$$

and:

$$\lim_{x\rightarrow Xn0^-}E\left(x\right)=0 \quad \Longrightarrow \quad E_0^-=-\frac{qN_d}{\epsilon_{Si}}X_{n0} \tag{13}$$

Still because of electric field continuity we can write:

$$\lim_{x\rightarrow 0^-}E\left(x\right)=\lim_{x\rightarrow 0^+}E\left(x\right) \quad \Longrightarrow \quad E_0^-=E_0^+=E_0 \tag{14}$$

And Therefore:

$$E_0=-\frac{qN_a}{\epsilon_{Si}}X_{p0}=-\frac{qN_d}{\epsilon_{Si}}X_{n0} \tag{15}$$

Eq. 15 also directly relates the dimensions of the space charge region on the left and right sides.

$$N_dX_{n0}=N_aX_{p0} \tag{16}$$

Note: Eqs. 15 and Eq. 16 are the direct consequence of the zero electric field outside the space charge region, and therefore of the overall charge neutrality of the PN junction. Therefore, Eq. 16 can be rewritten more quickly by directly applying the charge neutrality principle. In this case, the total amount of charge in the P-Type part is:

$$Q_p=-qN_aX_{p0} \tag{17}$$

And the total amount of charge in the N-Type part is:

$$Q_n = qN_dX_{n0} \tag{18}$$

Now, the charge neutrality condition is:

$$Q_n + Q_p = 0 \tag{19}$$

and finally:

$$N_dX_{n0} = N_aX_{p0} \tag{20}$$

Note: By construction in Eq. 11, the integration constant E_0 represents the electric field value at $x = 0$, which also corresponds to the maximum value of the electric field throughout the PN junction.

Solution D

The electrical potential across the structure is depicted in Fig. 8. This potential can be easily calculated using the formula:

$$\phi(x) = -\int_{-\infty}^{x} E(x)\,dx \tag{21}$$

To simplify the integration, we will quickly rewrite the electric field in Eq. 11 using the definition of E_0 given in Eq. 15:

$$E(x) = \begin{cases} 0 & , \quad x \in]\leftarrow; -X_{p0}] \\ -\frac{qN_a}{\epsilon_{Si}}(x + X_{p0}) & , \quad x \in]-X_{p0}; 0] \\ \frac{qN_a}{\epsilon_{Si}}(x - X_{n0}) & , \quad x \in]-0; X_{n0}] \\ 0 & , \quad x \in]X_{n0}; \rightarrow] \end{cases} \tag{22}$$

We can utilize Eq. 21 to determine the electrical potential across the structure. Currently, we cannot assume anything about the electrical potential of the P-Type and N-Type regions; therefore, we will use ϕ_1 and ϕ_2 respectively.

$$\phi(x) = \begin{cases} \phi_1 & , \quad x \in]\leftarrow; -X_{p0}] \\ \phi_1 + \frac{qN_a}{\epsilon_{Si}}(x + X_{p0})^2 & , \quad x \in]-X_{p0}; 0] \\ \phi_2 - \frac{qN_d}{\epsilon_{Si}}(x - X_{n0})^2 & , \quad x \in]-0; X_{n0}] \\ \phi_2 & , \quad x \in]X_{n0}; \rightarrow] \end{cases} \tag{23}$$

Due to the integration of the electric field, we can guarantee that the electric potential is continuous. Therefore:

$$\lim_{x \to 0^+} \phi(x) = \lim_{x \to 0^-} \phi(x) \tag{24}$$

$$\phi_1 + \frac{qN_a}{\epsilon_{Si}} X_{p0}^2 = \phi_2 - \frac{qN_d}{\epsilon_{Si}} X_{n0}^2 \tag{25}$$

That we can rewrite:

$$\frac{q}{\epsilon_{Si}} \left(N_d X_{n0}^2 + N_a X_{p0}^2 \right) = \phi_2 - \phi_1 = \phi_b \tag{26}$$

Solution E

To guaranty potential continuity, Eq. 26 provides us with the potential difference between the two parts of the structure. As we have already calculated in Eq. 7, this corresponds to the built-in potential ϕ_b. In Eq. 16, we have established a relationship between X_{n0} and X_{p0}. In this example, we decide to express X_{n0} as a function of X_{p0}:

$$\frac{q}{2\epsilon_{Si}} N_a \frac{N_a + N_d}{N_d} X_{p0}^2 = \phi_b \tag{27}$$

We can now extract X_{p0}:

$$X_{p0} = \sqrt{\frac{2\epsilon_{Si} N_d \phi_b}{qN_a \left(N_a + N_d \right)}} = 298 \, [nm] \tag{28}$$

Still, by utilizing the relationship between X_{n0} and X_{p0}, we can write:

$$X_{n0} = \sqrt{\frac{2\epsilon_{Si} N_a \phi_b}{qN_d \left(N_a + N_d \right)}} = 29.8 \, [nm] \tag{29}$$

Note: We can observe from Eq. 16 that, due to overall charge neutrality, the p-type and n-type depletion lengths are linked. Here, it is evident that the n-type depletion length is 10 times smaller than the p-type depletion length, indicating that the acceptor concentration is 10 times greater than the donor concentration.

Note: Be very careful with the units. In semiconductor physics, we prefer to work in centimeters $[cm]$ rather than meters $[m]$. Therefore, you must ensure that everything is expressed in $[m]$. For example, in this series, we provide the vacuum and silicon dielectric constants expressed in $[F/cm]$, but often, such constants are given in $[F/m]$ on calculators or the internet. This also implies that distance results, such as depletion length, will be given in $[mm]$, and you will need to convert them back to $[m]$.

To Go Further

To properly complete the PN-junction calculation, we will determine the maximum electric field E_0. To do this, we will simply substitute the formula for X_{p0} found in Eq. 28 into the equation for E_0 given in Eq. 15. This gives us the following equation:

$$E_0 = \sqrt{\frac{2q}{\epsilon_{Si}} \frac{N_a N_d}{N_a + N_d} \phi_b} = 4.6 \left[\frac{MV}{m}\right] \tag{30}$$

Note: Again be very careful with the units.

Exercise 03

Considering the junction calculated in the preceding exercise with depletion approximation, if Q is the charge in the n-Type part of the space charge region, and $-Q$ is the charge in the p-Type part of the space charge region, calculate $\frac{\partial Q(\phi_b)}{\partial \phi_b}$.

Solution

With the depletion approximation we have:

$$Q = qN_d X_{n0} \tag{31}$$

And as calculated in the preceding exercise:

$$X_{n0} = \sqrt{\frac{2\epsilon_{Si} N_a \phi_b}{qN_d \left(N_a + N_d\right)}} \tag{32}$$

This give us:

$$Q = \sqrt{2q\epsilon_{Si} \frac{N_a N_d}{N_a + N_d}} \sqrt{\phi_b} \tag{33}$$

And therefore, the derivative is:

$$\frac{\partial Q\left(\phi_b\right)}{\partial \phi_b} = \sqrt{2q\epsilon_{Si} \frac{N_a N_d}{N_a + N_d}} \frac{1}{2\sqrt{\phi_b}} \tag{34}$$

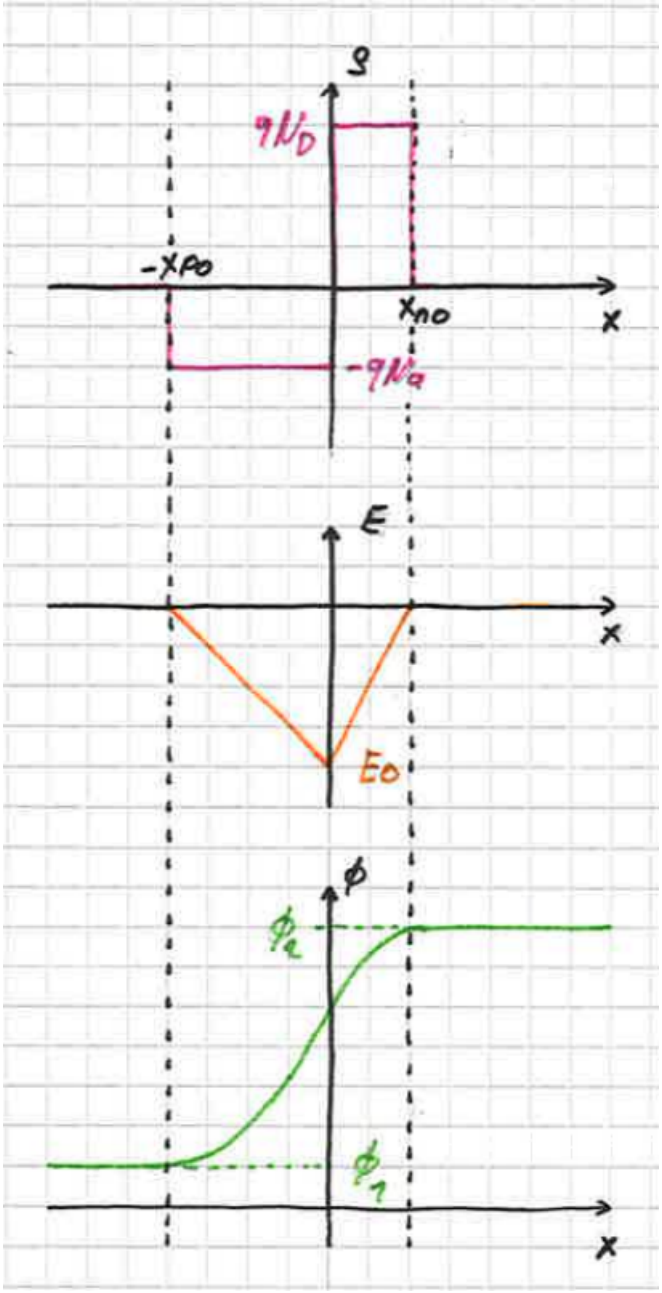

Figure 8: Illustration of different physical values through the PN-junction: A) Charge density distribution in the PN-junction. B) Electric field distribution in the PN-junction. C) Potential distribution in the PN-junction.